Crossing Probabilities and ARAIM Temporal Correlation

Carl Milner, Eugene Bang, Christophe Macabiau

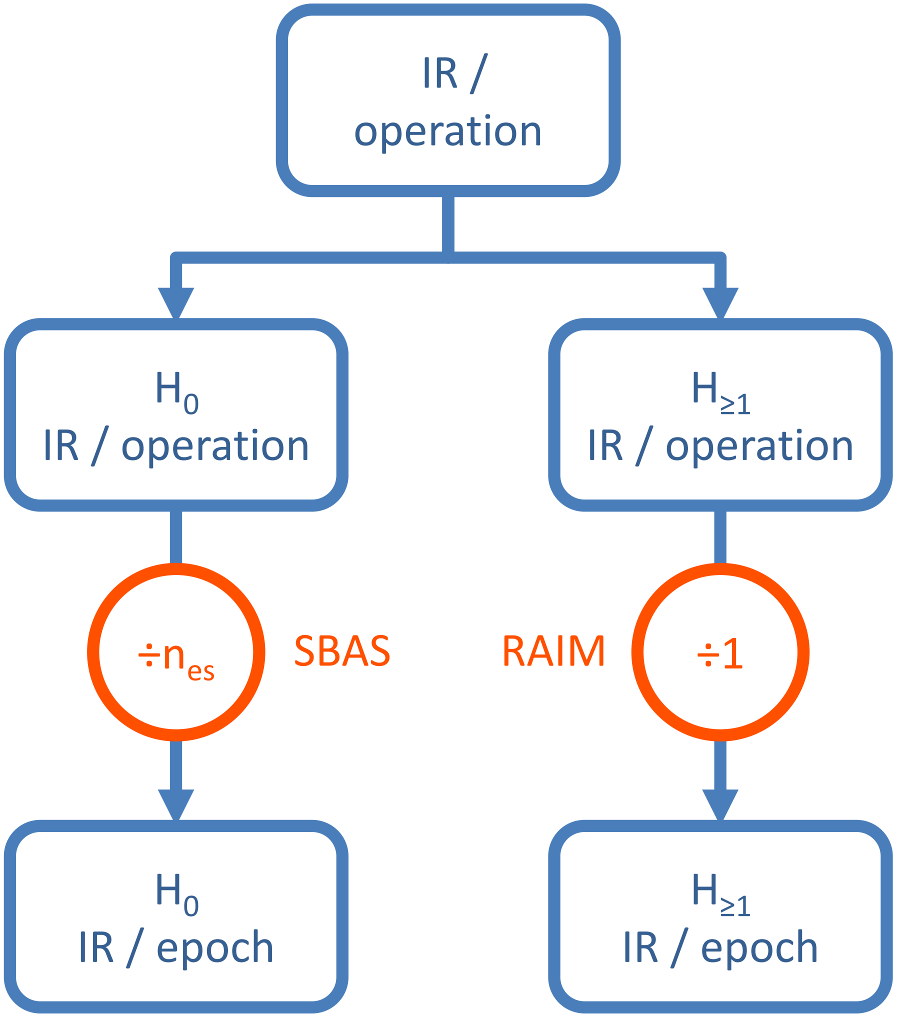
Integrity Allocation : 1/4

Allocation from the operational requirement to the per epoch requirement for the receiver

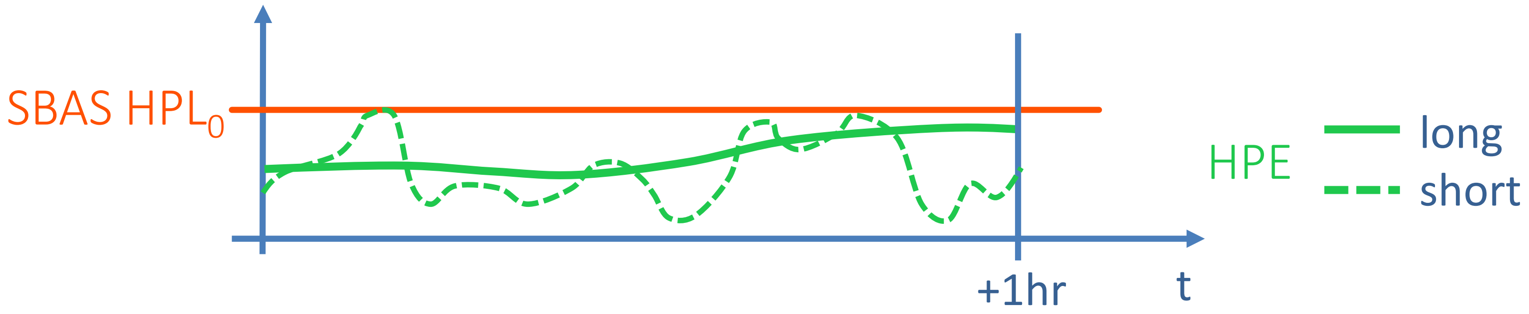
IIT work [1] on temporal correlation raised the question of whether this allocation has been correctly performed in the past *and* how it might be made for ARAIM

ARAIM is more complex since the allocation between H_0 (fault-free) and $H_{\geq 1}$ (faulty) is dynamic (geometry, ISM)

Gauss-Markov autocorrelation function assumed for all quantities (gradient of 0.9902 as in [1])



Integrity Allocation : 2/4



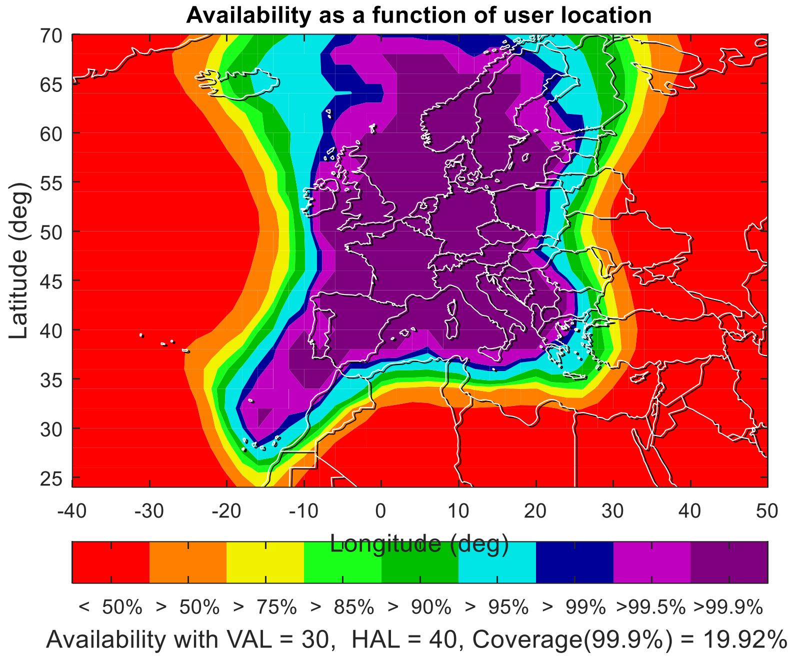
- Shorter time constant leads to more effective samples n_{es} resulting in higher integrity risk within the given duration

- The current assumption for SBAS integrity using iono correlation of 6 minutes [2]
  - 1 hour / 360 sec = 10 *independent* samples for NPA

- Actual integrity risk can be estimated (assuming stationary geometry) as
  - $IR_{operation} = n_{es} \times IR_{epoch}$

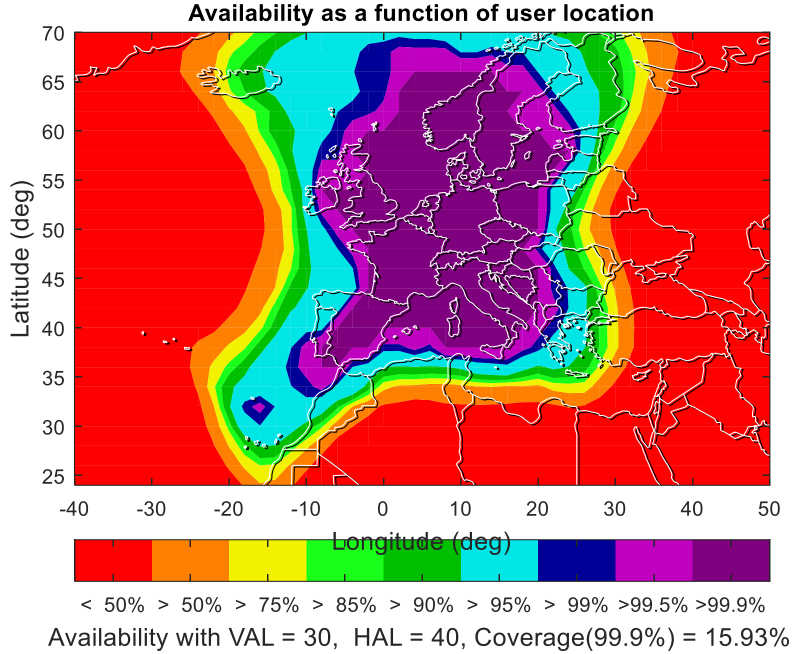
- Leads to effective SBAS protection level inflation ≈ 1.1-1.2 over the existing method

Integrity Allocation : 3/4



LPV-200 Baseline availability

K_V=5.33, K_H=6.00



LPV-200 Availability with new Ks

K_V=5.88, K_H=6.75

(~10% increase in both K_V K_H)

Parameter	Description
Constellation	MOPS 24 GPS L1 C/A (RTCA, 2009)
GIVE	GIVE variance model (ESA SBAS simulator) / MOPS interpolation
UDRE	UDRE variance model with MT27 (ESA SBAS simulator)
Time step	24 hours (10 min step)

Integrity Allocation : 4/4

For ARAIM, more complex since [3]:

$$P_{hmi} = P(hmi|H_0) + \sum_{j=1}^{N_h} P\left(hmi\,\middle|H_j\right) P(H_j)$$

Effective number of samples $n_{es,0}$ for $P(hmi|H_0)$ expect similar to (DFMC) SBAS HPL_0

Modification? $P_{hmi} = n_{es,0}P(hmi|H_0) + \sum_{j=1}^{N_h} n_{es,j}P\left(hmi\,\middle|H_j\right) P(H_j)$

Yet H_0 is not the dominant fault mode, so how to manage n_{es}?

$P(H_j)$ - prior prob. fault mode

$P\left(hmi\,\middle|H_j\right)$ - prob. HMI given fault mode

Reinclusion : 1/2

Option 1 : *instant reinclusion*

A previously excluded satellite is reincluded and tested every epoch

start below

start above ∩ cross below

q is the test statistic
k is the threshold

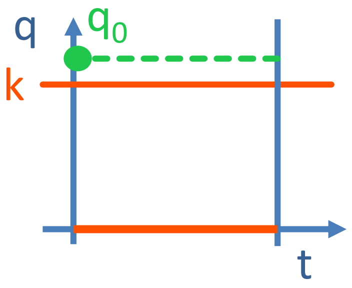
As far as we are currently the baseline in ARAIM (...and RAIM)

\+ **availability** over **integrity**

Reinclusion : 2/2

Option 2 : *operational exclusion*

A satellite is excluded for the length of the period of operation (1hr or 150s)



start above (excluded)

\+ **integrity** over **availability/continuity**

Integrity Requirement : 1/2

Integrity Risk requirement is in any hour or approach [4], meaning:

$$P\left(\bigcup_i e_i > l \cap q_i < k\right)$$

q is the test statistic
k is the threshold
e is the pos. error
l is the prot. level

i is the discrete time sample index

The TTA condition is neglected in the notation above but also plays a role [1]

Explicitly:

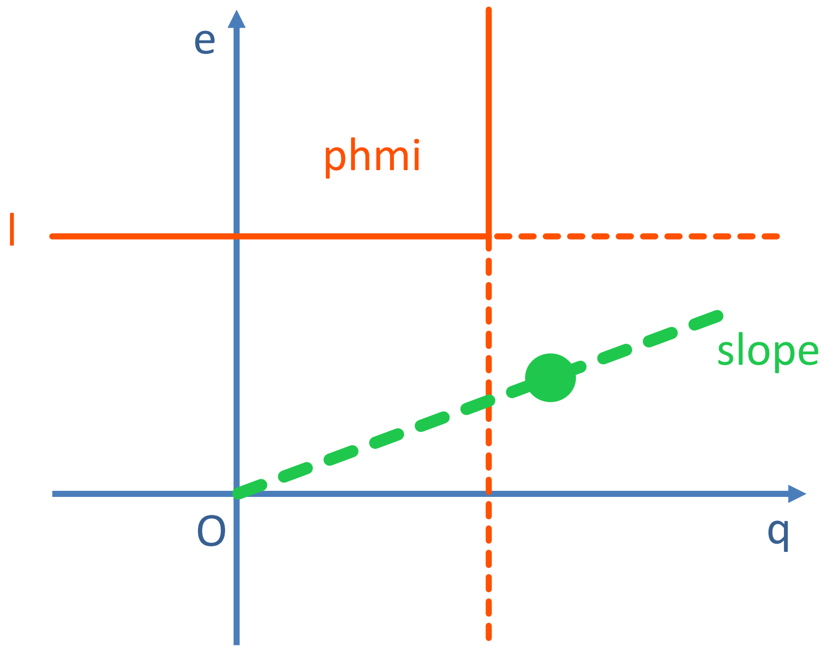
$$P\left(\bigcup_i e_i > l \cap q_i < k \,\middle|\, H_j\right) = \max_{\boldsymbol{\mu}(t)} P\left(\bigcup_i e_i > l \cap q_i < k \,\middle|\, H_j, \boldsymbol{\mu}(t)\right)$$

Where $\boldsymbol{\mu}(t)$ is the fault vector profile over time in the range domain

Integrity Requirement : 2/2

Conjecture: The worst case probability $\max_{\boldsymbol{\mu}} P\left(\cup_i e_i > l \cap q_i < k \,\middle|\, H_j, \boldsymbol{\mu}\right)$ is given for a constant fault profile i.e. $\boldsymbol{\mu}(t) = \boldsymbol{\mu}$

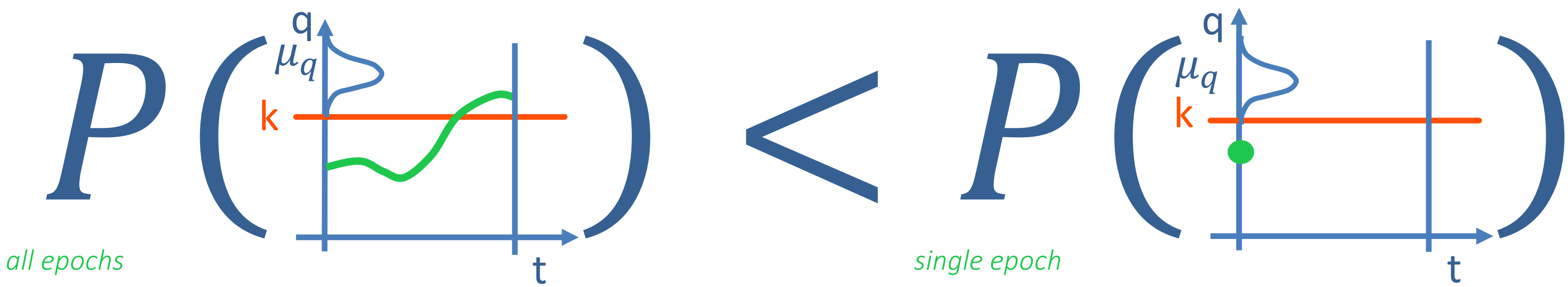
This conjecture is assumed true for the remainder of the work presented



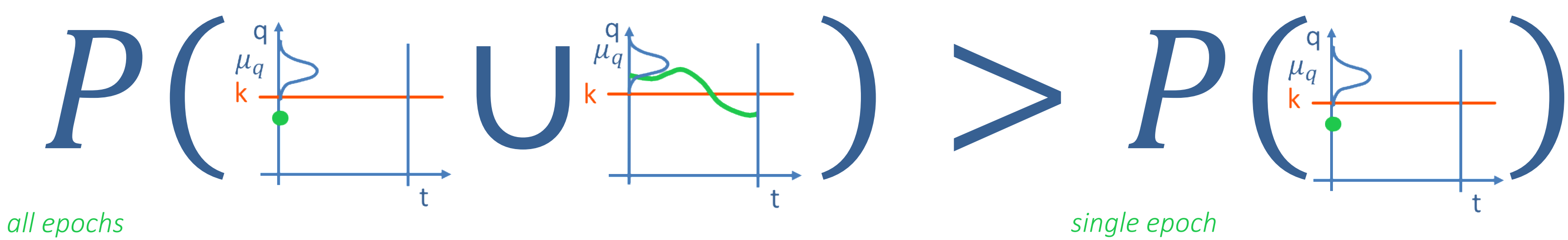
Instant Reinclusion

Instant Reinclusion Integrity: 1/6

Traditional argument concerning test statistic q [1][5]

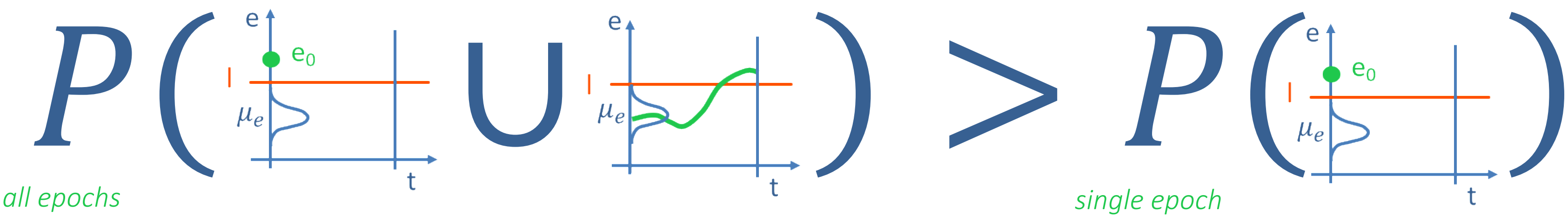


However, with ***instant reinclusion*** the case of start above and cross below is added:

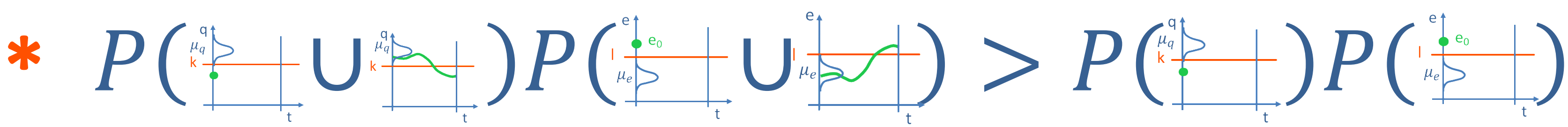


Instant Reinclusion Integrity: 2/6

Even if the no alert condition bounding were valid, for the positioning failure



Combining the bounds with *instant reinclusion*:



e is the position error

l is the protection level

Note:

1. Right hand side is bounded by current protection level
2. Left hand side is conservative since simultaneity of boundary crossing neglected

Instant Reinclusion Integrity: 3/6

probability of hazardously misleading information

$$P\left(\bigcup_i e_i > l \cap q_i < k\right) \quad samples\ i$$

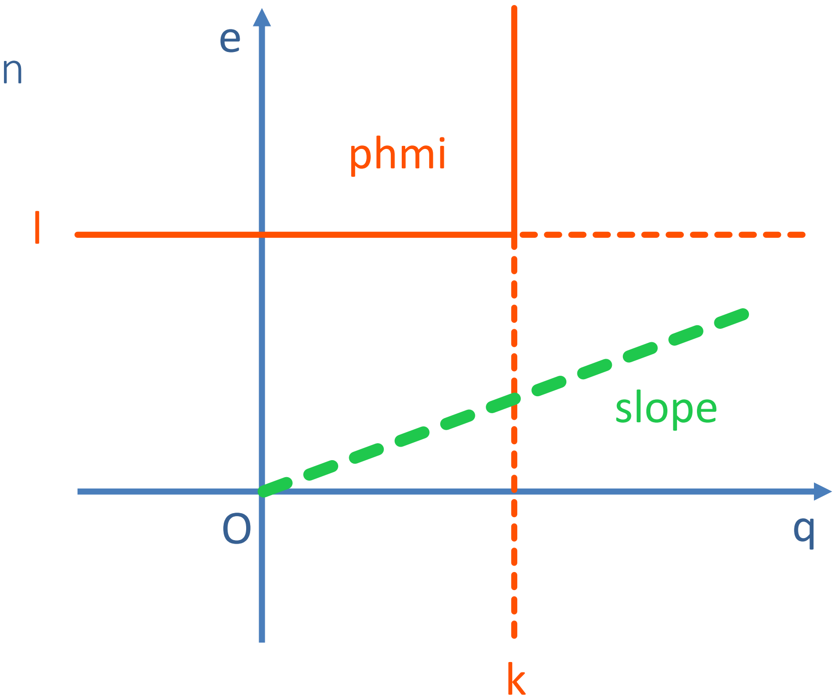
where:

q is the normalised test statistic

k is the normalised threshold (k factor)

e is the normalised position error

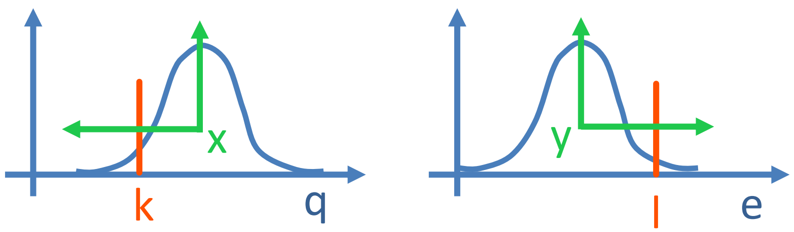
l is the normalised protection level



Transform for each fault magnitude μ ($\boldsymbol{\mu} = \mu \boldsymbol{u}_{wcd}$ [6][7]) with equivalent in the test μ_q and position μ_e :

$$q \rightarrow q - \mu_q = -x \quad k - \mu_q = -m_x$$

$$e \rightarrow e - \mu_e = y \quad l - \mu_e = m_y$$



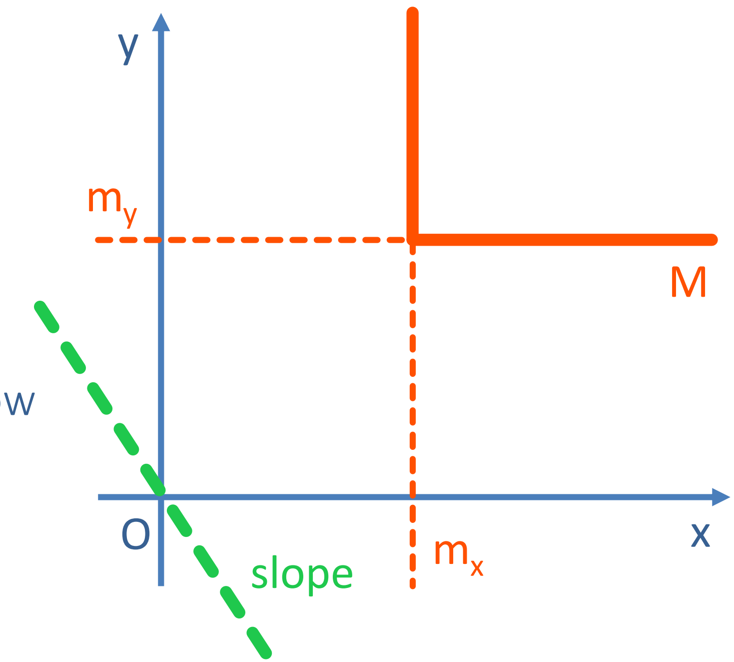
worst case bias situation

Instant Reinclusion Integrity: 4/6

Reformulated then as:

$$1 - P_{<M}(N) = P\left(\bigcup_i x_i > m_x \cap y_i > m_y\right)$$

$P_{<M}$ is the probability that the process starting below a threshold M does not exceed it by sample N

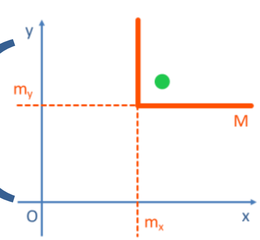


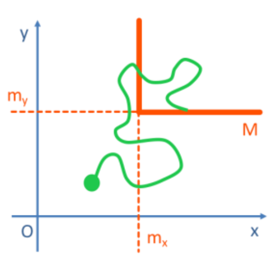
Related to the maximum of the process

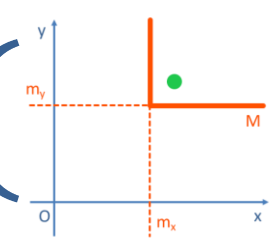
Can update *

all epochs

single epoch

+ $P\Big($ 

$\cup$ 

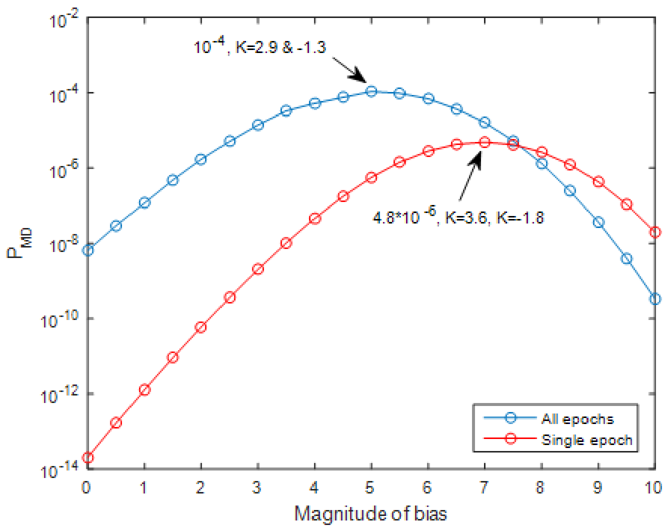
$\Big) > P\Big($ 

$\Big)$

Instant Reinclusion Integrity: 5/6

Effective number of samples n_j for fault mode H_j given as follows [1]:

$$\frac{\max_{\mu} P\left(\cup_i e_i > l \cap q_i < k \middle| H_j, \mu\right)}{\max_{\mu} P\left(e_0 > l \cap q_0 < k \middle| H_j, \mu\right)}$$

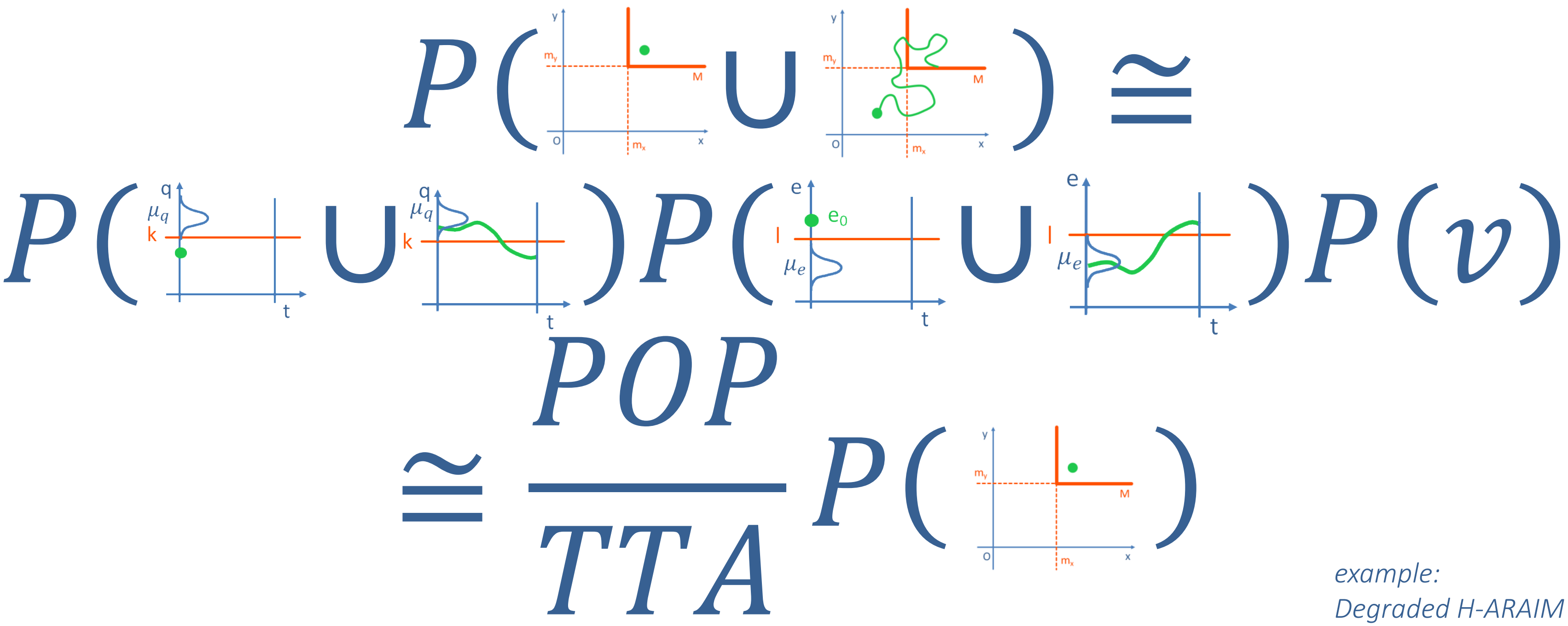


Note:

1. The maximum over the fault bias may be at different values of fault magnitude
2. The worst case fault direction is determined for the denominator
3. The true risk should account for variable fault profile
4. The variation of the constellation not a major issue
5. The SS protection level already bounds the denominator but this margin is not known in advance

Instant Reinclusion Integrity: 6/6

Modelled the probability based on the maximum of a GM process over the operational period (1 hour) and an overlap function $P(v)$



example:
Degraded H-ARAIM geometry

hpl	*= 127*
k_fa	*= 5.63*
slope	*= 12.55*
sigma	*= 1.5*
P(all epoch)	*= 1.5e-4*
P(1 epoch)	*= 1.5e-6*
P(pl alloc)	*= 3.9e-4*

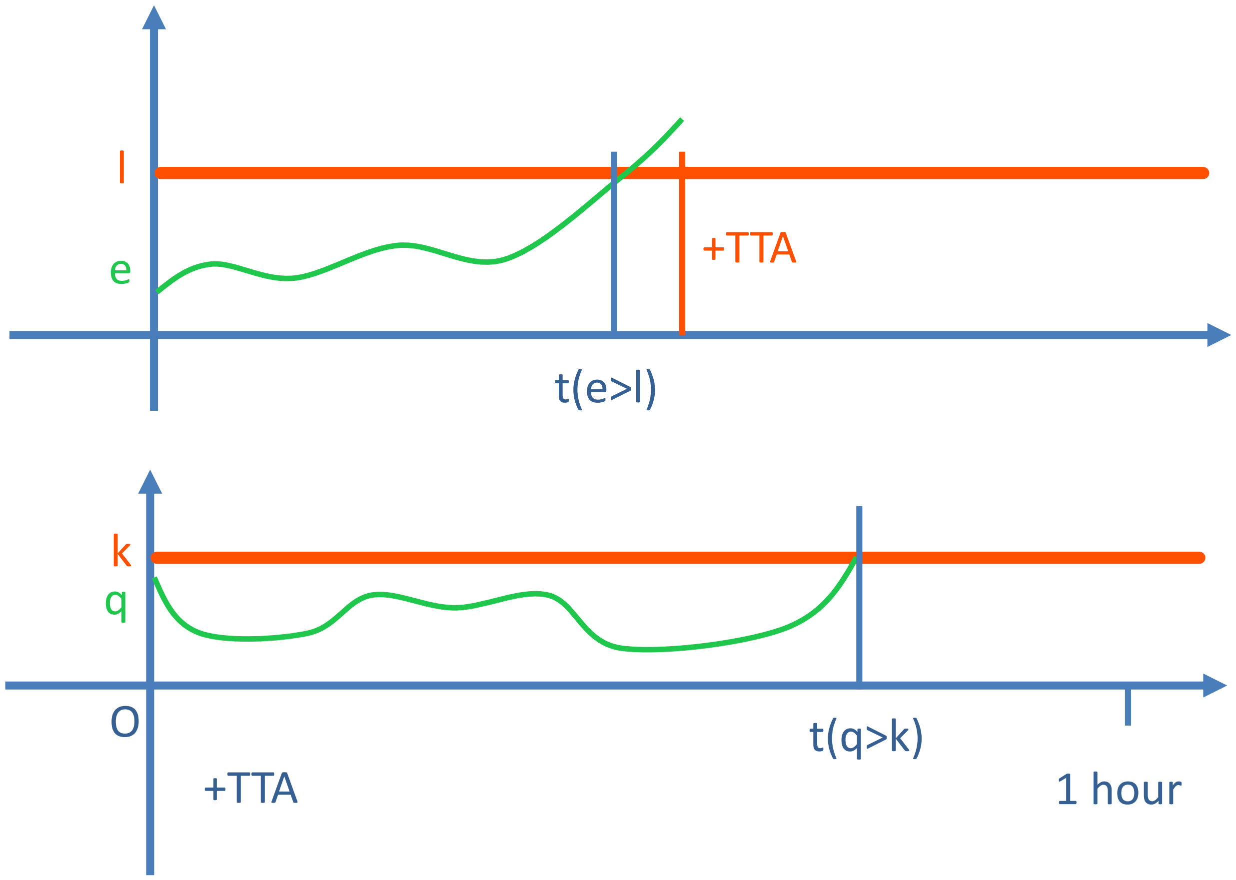
Summary : 1

Results suggest that it may be necessary to mitigate the potentially large number of effective samples:

- accept the need for longer exclusion (e.g. operational exclusion)
- justify a lower prior propability for the worst case
- justify the use of the allocated probability to assess the effective risk

Operational Exclusion

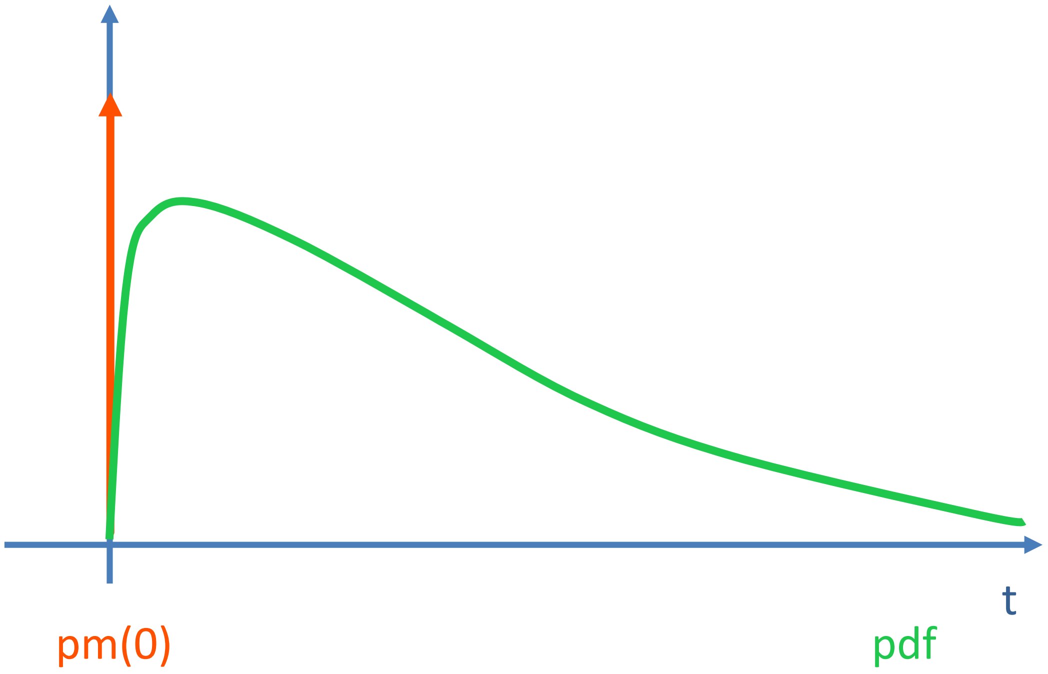
Operational Exclusion Integrity: 1/3



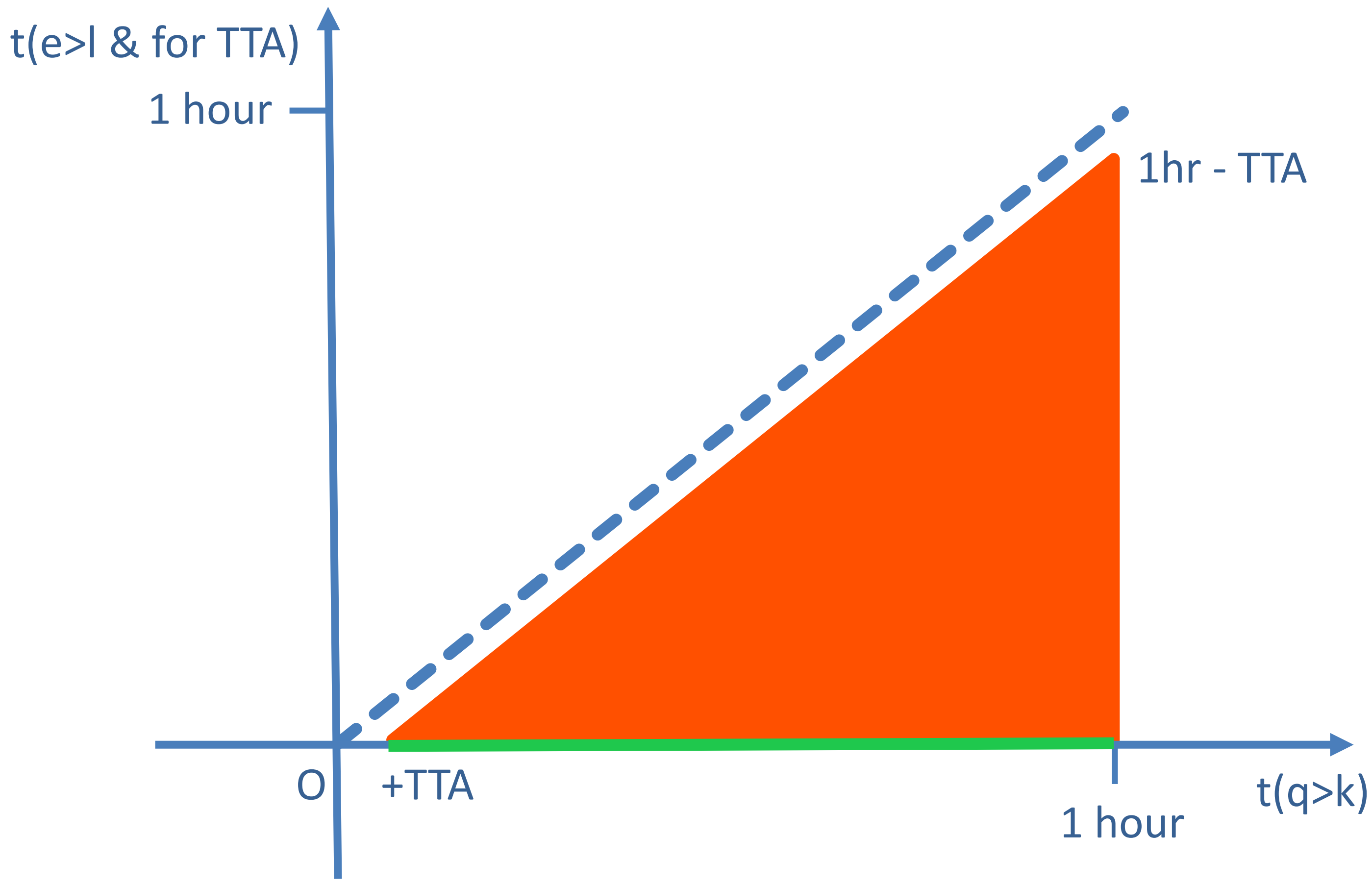
Probability distributions for the time of first passage of a boundary are needed (theoretical semi analytic approach or empirical approach)

Operational Exclusion Integrity: 2/3

First passage time density for a starting value below a boundary and a delta at 0 relating to the probability mass of beginning



Operational Exclusion Integrity: 3/3



Number of effective samples below but close to one.

Summary: 2

Initial results appear to show low value for the effective number of samples in the **operational exclusion** case as expected (TBC).

Remains to be determined the impact on availability and continuity

Presents one possible solution to the temporal correlation problem.

Next Steps : 1

Complete implementation of 1D process using theoretical development

Complete theoretical extension to 2D quadrant boundary crossing problem (pure simultaneity accounted)

Determine if the protection level can be proven to overbound the all epochs risk

Consider the impact of time varying bias or prove the conjecture

Consider the impact of the sum of multiple GM processes

Check the empirical cross correlation between test statistic and position error

References

[1] Pervan, B. *et al* (2017) "ARAIM Fault Detection and Exclusion", ITSNT 2017, Toulouse, France, November 2017

[2] Roturier et al., "The SBAS integrity concept standardised by ICAO: Application to EGNOS" The Journal of Navigation, Vol.49, no196, 2001

[3] Blanch, J. *et al*. (2012) Baseline Advanced User Algorithm and Possible Improvements

[4] ICAO Annex 10 (2006) SARPs

[5] Lee, Y. *et al* (1996) Summary of RTCA SC-159 GPS Integrity Working Group Activities, ION NTM

[6] Joerger, M. *et al* (2014) Solution Separation vs Residual-Based RAIM

[7] Angus, J. (2006) RAIM with Multiple Faults, NAVIGATION

[8] DeCleene, B. (2000) Defining Pseudorange Integrity - Overbounding

[9] Capinski and Kopp (1999) Measure, Integral and Probability, Springer

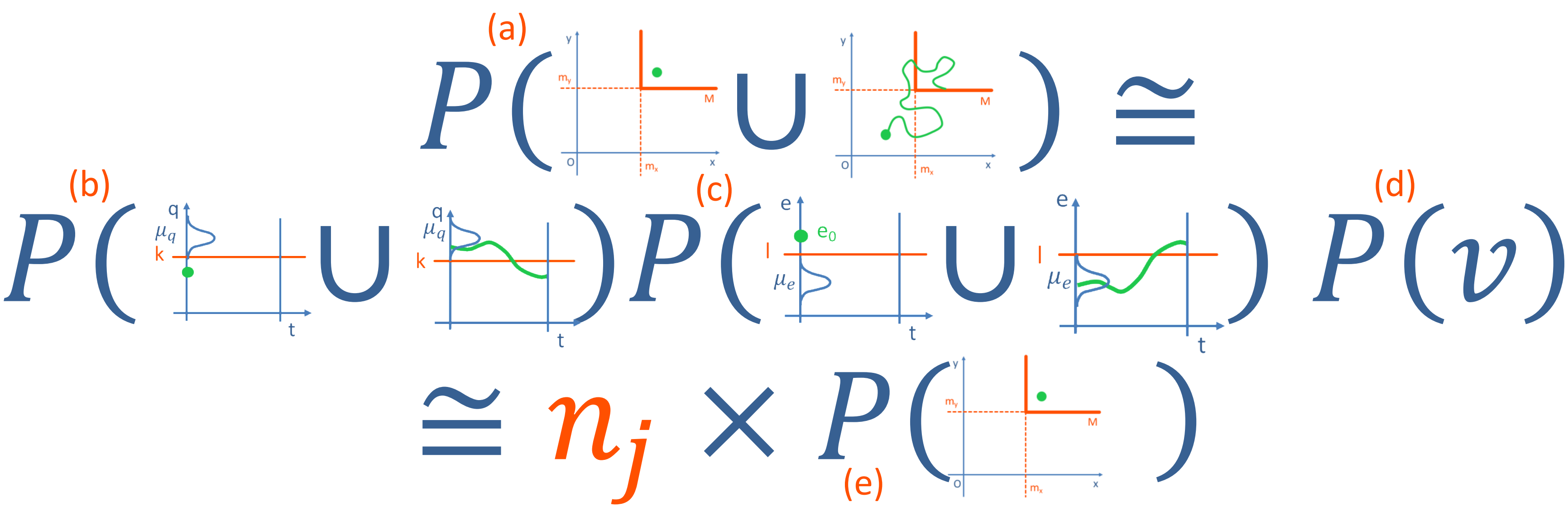
Questions…?

Backup Slides

Crossing Probabilities and ARAIM Integrity Risk

Accounting for temporal correlation when the reinclusion principle is used could mean that the PHMI during the operation (i.e. over all epochs) (a) could exceed the PHMI for a single epoch (e) by a factor of n_j.

Or as below: the PHMI over all samples is approximately equal to the product of the probability of no detection over all samples (b), the probability of a positioning error (c) over all samples and the overlap probability (d)

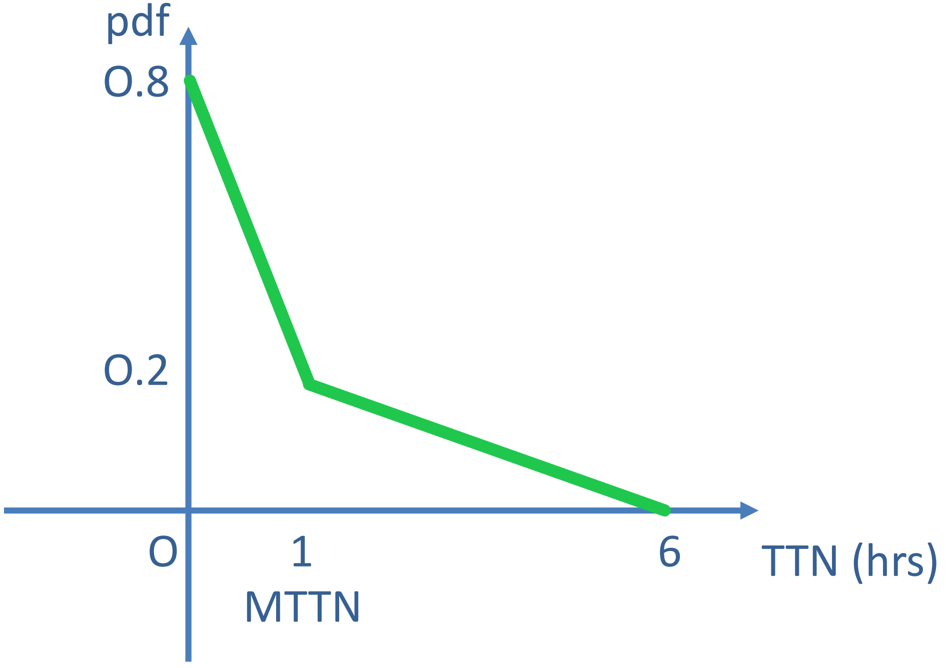


Integrity Requirement : 3

The prior probability of failure must be determined for faults which last the total exposure time of 1 hour, the onset probability P_{onset} for GPS is $\frac{10^{-5}}{hour}$ with a maximum and mean TTN of 6 hours and 1 hour respectively.

Using the mean and emplying linear pdf over the two intervals [0, 1] and [1, 6] the following prior probability is obtained

$$P_{fail\,\geq 1hr} = \frac{10^{-5}}{hour} \times 0.2 \times 5 \times 0.5 = 5 \times 10^{-6}$$



Integrity Requirement : 4

The use of the full exposure time of 1 hour is not enitrely evident. However, it is clear that the maximum HMI is observed for this case since the fault probability diminishes slower than the conditional HMI as a function of exposure time **ΔT** if **n_{es}** is larger than 2 and the function is not significantly non-linear*