

# **Worst Impact of Pseudorange nominal Bias on the Position in a Civil Aviation Context**

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ITSNT 2017, Toulouse

November 16<sup>th</sup>

## What is a pseudorange nominal bias?

GNSS signals are stained by errors even in the nominal case:

### Satellite

- Payload
- Antenna

### Propagation medium

### Receiver

- Antenna
- Radio-Frequency front-end.

Tracking process to estimate pseudoranges from signals

induces additional errors.

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Pseudoranges nominal errors.

2 kinds of pseudorange errors  
[Macabiau et al., 2014]

Random error	<b>Bias</b>
Distribution of the summed error overbounded by $N(0, \sigma^2)$ .	Very long term variation
Short term variation	

$N(0, \sigma^2)$  is the Gaussian distribution with a zero mean and a standard deviation equal to  $\sigma$ .

### Bias and Civil Aviation

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### Bias and Civil Aviation

**Question:** Why is it important to make the difference between a random error and a bias?

**Answer:** The difference is important in a civil aviation context:

- It affects the way the error sources are incorporated into the aircraft position protection level calculations, an important tool related to integrity [Raghuvanshi and Van Graas, 2015].
- A precise knowledge of nominal errors is important to define the limit between the nominal and the non-nominal cases. It improves integrity.

### From pseudorange biases to position biases



- Pseudorange biases can entail position bias.
- To introduce a problem in the position domain, biases have to be different on each pseudorange measurement (otherwise biases are absorbed in the time estimated by the receiver).
- Depending on the PVT algorithm, same pseudorange biases may lead to different PVT solution. A least square algorithm (not weighted) is applied in the publication (commonly used in civil aviation).
- The sum of phase pseudorange nominal biases is below 20 cm. In civil aviation, the code pseudorange smoothing reduces the impact of phase pseudorange biases
  - ⇒ phase measurements biases are neglected in the following.

### Aims of this publication

- Propose a model to **characterize code pseudorange nominal bias** that affects traditional **civil aviation GPS L1 C/A** receiver.
- **Estimate the impact of code pseudoranges biases in the position domain** based on a Matlab software. This impact is assessed for different locations around the world.
  - As the position error is dependent upon several parameters, a **worst case** among the different parameters is looked at.

- I. Definitions related to pseudorange nominal bias
- II. Pseudorange nominal bias model
- III. Simulation concept
- IV. Results: impact of pseudorange nominal biases on the position
- V. Conclusions and future works

## **I. Definitions related to pseudorange nominal bias**

II. Pseudorange nominal bias model

III. Simulation concept

IV. Results: impact of pseudorange nominal biases on the  
position

V. Conclusions and future works

# I. Definition related to pseudorange nominal bias (1/2)

## 1) Code pseudorange model

$$P^{Xi} = \sqrt{(x - x^i)^2 + (y - y^i)^2 + (z - z^i)^2} + I^i + T^i + \text{clock\&eph}^{Xi} + \text{mult}_\rho^i + n_\rho^i + b_\rho^X + b_\rho^{Xi}$$

Range between the  $i^{\text{th}}$  satellite position  $(x^i, y^i, z^i)$  and the receiver position  $(x, y, z)$

$X = \text{L1 or L5 (GPS), E1 or E5 (GALILEO)}$ .

In this model, the range residual quantities involved are:

- $\text{clock\&eph}^{Xi}$  is the sum of residual range error due to ephemeris error and satellite clock error.
- $T^i, I^i, \text{mult}_\rho^i, n_\rho^i$  are the residual tropospheric, ionospheric and multipath plus noise errors.
- $b_\rho^X$  is the receiver clock offset with respect to the constellation reference time (common to all pseudoranges).

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Traditionally considered as **random errors** (ex:ARAIM)

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Traditionally considered as **random errors** (ex:ARAIM)

Only impact the **time** component.

# I. Definition related to pseudorange nominal bias (1/2)

## 1) Code pseudorange model

$$P^{Xi} = \sqrt{(x - x^i)^2 + (y - y^i)^2 + (z - z^i)^2} + I^i + T^i + \text{clock\&eph}^{Xi} + \text{mult}_\rho^i + n_\rho^i + b_\rho^X + b_\rho^{Xi}$$

➤  $b_\rho^{Xi}$  is the code pseudorange iono-free nominal biases for satellite  $i$  with a **long-term variation**.

Impact the **position** component.

This parameter is the important one when looking at the impact of pseudorange measurements in the position domain:

The bias that is different depending on code pseudorange measurements.

## 2) Pseudorange bias model

From the state-of-the-art, nominal bias can be split in 3 components (more details in the paper): delays induced by the satellite antenna, delays induced by the receiver antenna, bias induces by distortions [Macabiau et al., 2014]:

$$b^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + b_{dist}^{Xi}(N, el, az)$$

- $b_{SV}^{Xi}(N)$  is the nominal bias induced by the **satellite antenna group delay** variation,
- $b_{ant}^{Xi}(el, az)$  is the nominal bias induced by the **receiver antenna group delay** variation.
- $b_{dist}^{Xi}(N, el, az)$  is the nominal bias component due to the **distortion**.
- $i$  corresponds to signal  $i$ .
- The nominal bias is dependent upon  $N$  (Nadir),  $el$  (elevation) and  $az$  (azimuth) in the receiver coordinates system.

This definition of  $b^{Xi}(N, el, az)$  is used in the following.

I. Definitions related to pseudorange nominal bias

**II. Pseudorange nominal bias model**

III. Simulation concept

IV. Results: impact of pseudorange nominal biases on the  
position

V. Conclusions and future works

### 1) Concept

The following model is considered to characterize code pseudorange nominal bias:

$$b^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + b_{dist}^{Xi}(N, el, az)$$

⇒ To characterize the bias  $b^{Xi}(N, el, az)$  that affects pseudorange in nominal conditions it is necessary to characterize the 3 bias components.

Aim of this section:

Present models used in the publication to characterize the 3 bias components.

A wide state-of-the-art permits to choose models.

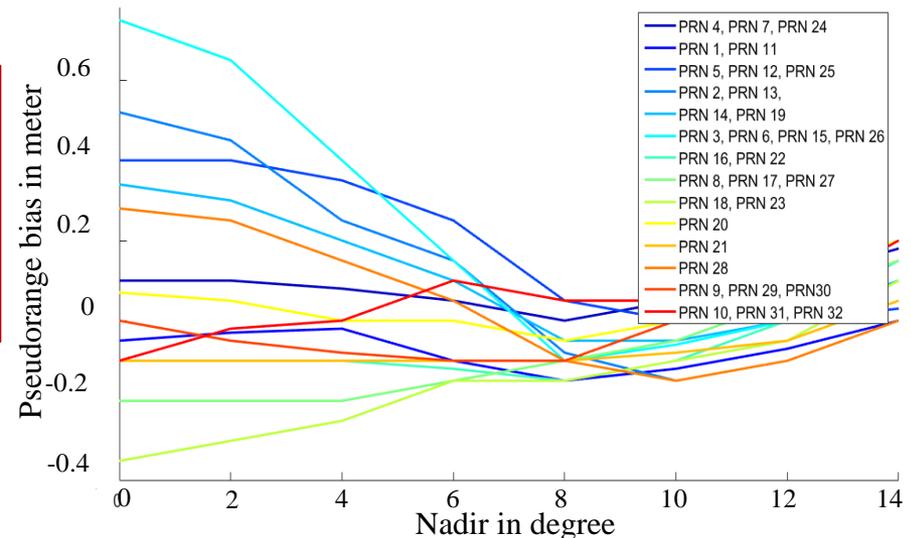
## II. Pseudorange nominal bias model (2/4)

### 2) Bias induced by the satellite antenna delay variation

$$b^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + b_{dist}^{Xi}(N, el, az)$$

- Different consistent publications considered. Results from [Haines et al., 2012] were used.
- When results are not available for some PRNs, results from other PRNs are used (Block IIR-M for II-F). Results given on L1/L2 iono-free measurements are used on GPS L1 C/A measurements.

For each satellite (PRN),  
the pseudorange error is given in meter  
function of the nadir angle ( $N$ ) in  
degree (every  $2^\circ$ ).  
It runs from -0.4 m to 0.7 m.

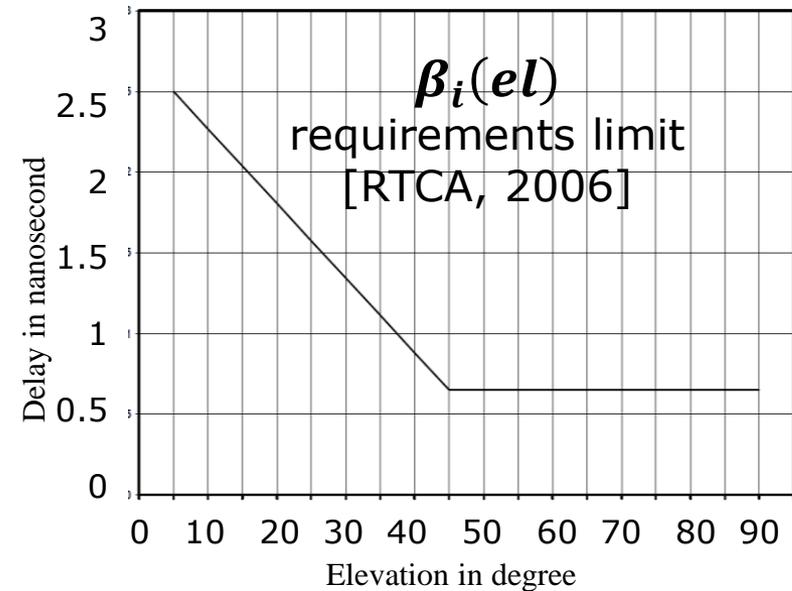
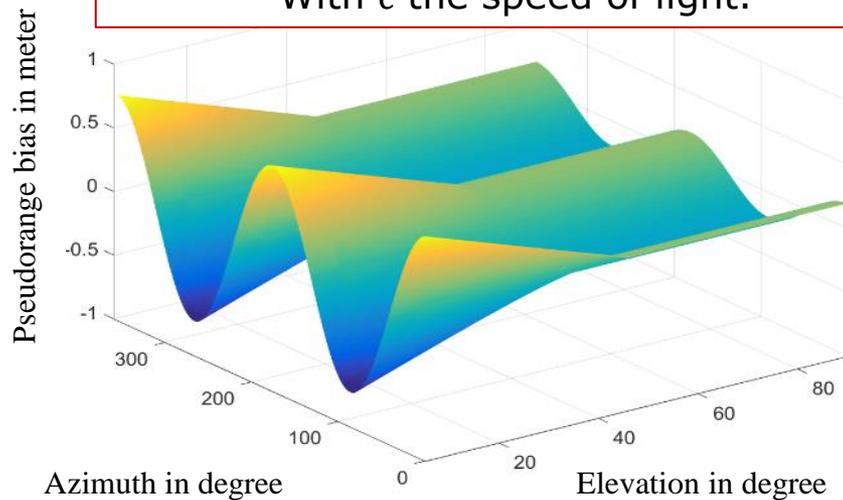


### 3) Bias induced by the receiver antenna delay variation

$$b^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + b_{dist}^{Xi}(N, el, az)$$

$$b_{ant}^{Xi}(el, az) = \beta_i(el) \times \sin(2az) \times c$$

With  $c$  the speed of light.



The conservative model from [Harris, 2016] is used (typical civil aviation receiver antenna in L1 band). A second model is tested in the publication.

Values from -0.75 m to 0.75 m.

### 4) Bias induced by distortions

$$b^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + \mathbf{b}_{dist}^{Xi}(N, el, az)$$

- Caused by electronic device imperfections at satellite and at receiver levels.
- No satisfying model was found to characterize the pseudorange bias induced by distortions.
- The concept is to consider that each (GPS L1 C/A) pseudorange measurement can be affected by a bias  $b_{max}$  with an absolute amplitude equal to 50 cm.

I. Definitions related to pseudorange nominal bias

II. Pseudorange nominal bias model

**III. Simulation concept**

IV. Results: impact of pseudorange nominal biases on the  
position

V. Conclusions and future works

### III. Simulation concept (1/2)

#### 1) From pseudorange biases to position bias

Position bias

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = S(k) \times \begin{bmatrix} b^1 \\ \vdots \\ b^N \end{bmatrix}$$

Pseudorange biases  
Depends upon the relative position between the satellite and the receiver antenna.

with  $S(k) = [H_{loc}^t H_{loc}]^{-1} H_{loc}^t(k)$

and

$$H_{loc} = - \begin{bmatrix} CE_1 CA_1 & CE_1 SA_1 & SE_i & -1 \\ \vdots & \vdots & \vdots & \vdots \\ CE_N CA_N & CE_N SA_N & SE_N & -1 \end{bmatrix}$$

where

$$\begin{aligned} CE_i &= \cos(el_i(k)) & SE_i &= \sin(el_i(k)) \\ CA_i &= \cos(az_i(k)) & SA_i &= \sin(az_i(k)) \end{aligned}$$

The geometry matrix  $H_{loc}$  depends on satellites (used in the PVT estimation)  $el$  and  $az$  in the receiver antenna coordinates system.

The relative positions between the receiver antenna and satellites antenna have an impact on the position bias.

### III. Simulation concept (1/2)

#### 1) From pseudorange biases to position bias

Position bias

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = S(k) \times \begin{bmatrix} b^1 \\ \vdots \\ b^N \end{bmatrix}$$

Pseudorange biases  
Depends upon the relative position between the receiver antenna and satellites

with  $S(k) = [H_{loc}^t H_{loc}]^{-1} H_{loc}^t(k)$

and

$$H_{loc} = \begin{bmatrix} CE_1 & CA_1 & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CE_N & CA_N & SE_N & SA_N & -1 \end{bmatrix}$$

where

$$CE_i = \cos(el_i(k))$$

$$SE_i = \sin(el_i(k))$$

$$CA_i = \cos(az_i(k))$$

$$SA_i = \sin(az_i(k))$$

To estimate the impact of pseudorange nominal biases on the position different satellite geometries and receiver locations have to be tested.

The geometry matrix  $H_{loc}$  depends on the relative positions of the satellites (used in the PVT estimation)  $el$  and  $az$  in the receiver antenna coordinates system.

The relative positions between the receiver antenna and satellites antenna have an impact on the position bias.

### 3) Simulations setup

- **Realistic orbital parameters** are considered to estimate the relative position between satellite antenna and the receiver antenna. Ephemeris are estimated from a **YUMA file** obtained the 04/02/2017. The constellation is based on 31 satellites, the satellite PRN 4 was not available.
- Number of **tested constellation geometries** considering one user: **720**, it corresponds to 1 test every 2 minutes during 24h ( $30 \times 24 = 720$ ).
- Number of **tested user locations**: **10 005** it corresponds to 69 values in latitude (from  $-85^\circ$  to  $85^\circ$  every  $2.5^\circ$ ) and 145 values in longitude (from  $-180^\circ$  to  $180^\circ$  every  $2.5^\circ$ ).
- Algorithm to estimate the position from pseudoranges: **least square** (not weighted).
- Satellite mask angle (to know which satellites are used to estimate the PVT):  **$5^\circ$** .

I. Definitions related to pseudorange nominal bias

II. Pseudorange nominal bias model

III. Simulation concept

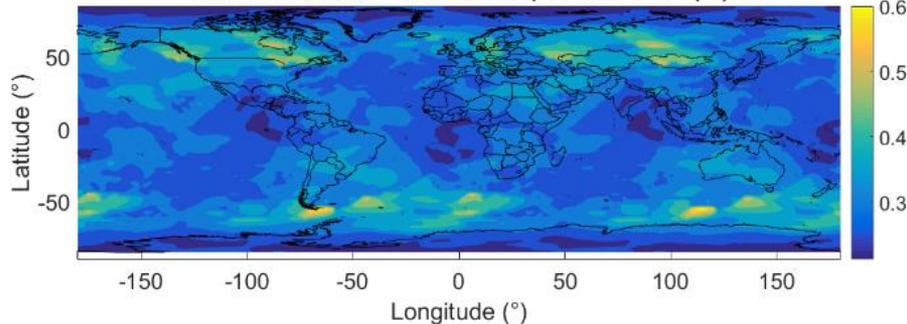
**IV. Results: impact of pseudorange nominal biases on  
the position**

V. Conclusions and future works

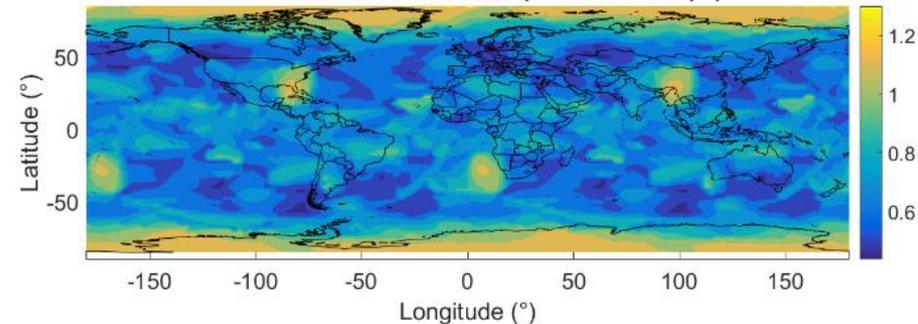
## IV. Results: impact of pseudorange nominal biases on the position (1/3)

### 1) Highest position biases induced by the satellite antenna delay variation

Maximum absolute horizontal position error (m)



Maximum absolute vertical position error (m)



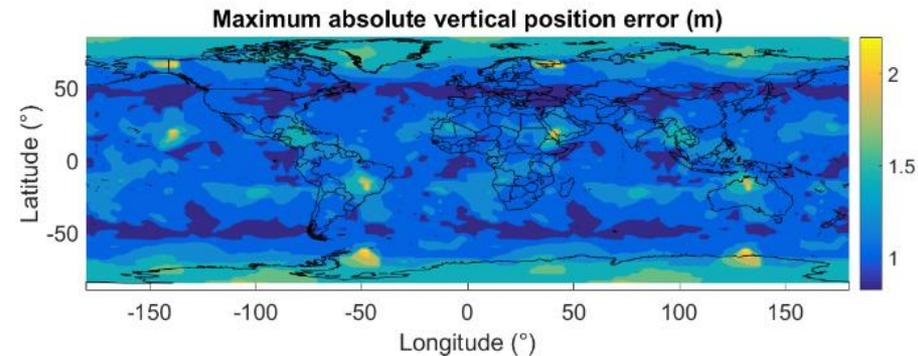
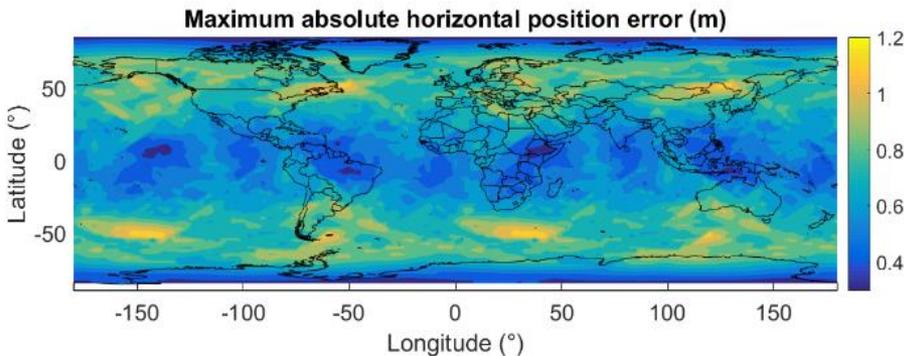
➤ Highest position biases among the 720 different epochs at the 10 005 different locations. It can be seen that:

- Horizontal position biases are small at the equator and at poles but can be high at mid-latitudes.
- Vertical position biases are small at the equator and at mid latitudes but are high at poles.
- The vertical position biases are in general higher than the horizontal position biases.

	Max All lat
Horizontal position	0.6 m
Vertical position	1.3 m
3D position	1.4 m

## IV. Results: impact of pseudorange nominal biases on the position (2/3)

### 2) Highest position biases induced by the receiver antenna delay variation



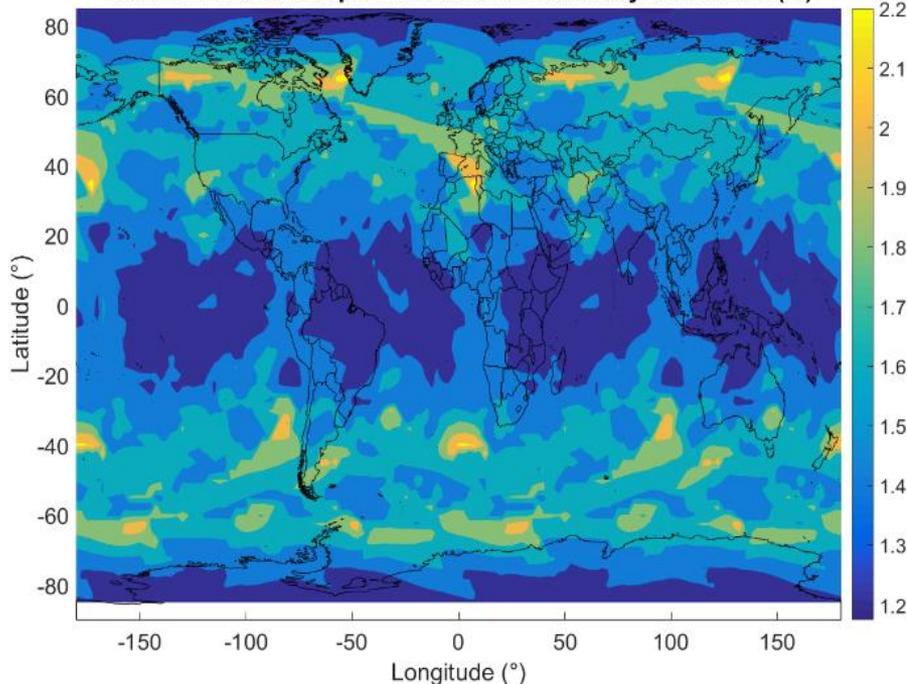
Worst case among different antenna orientations (different azimuths:  $0^\circ$ ,  $10^\circ$ , ...,  $80^\circ$ )

	Max All lat
Horizontal position	1.2 m
Vertical position	2.3 m
3D position	2.3 m

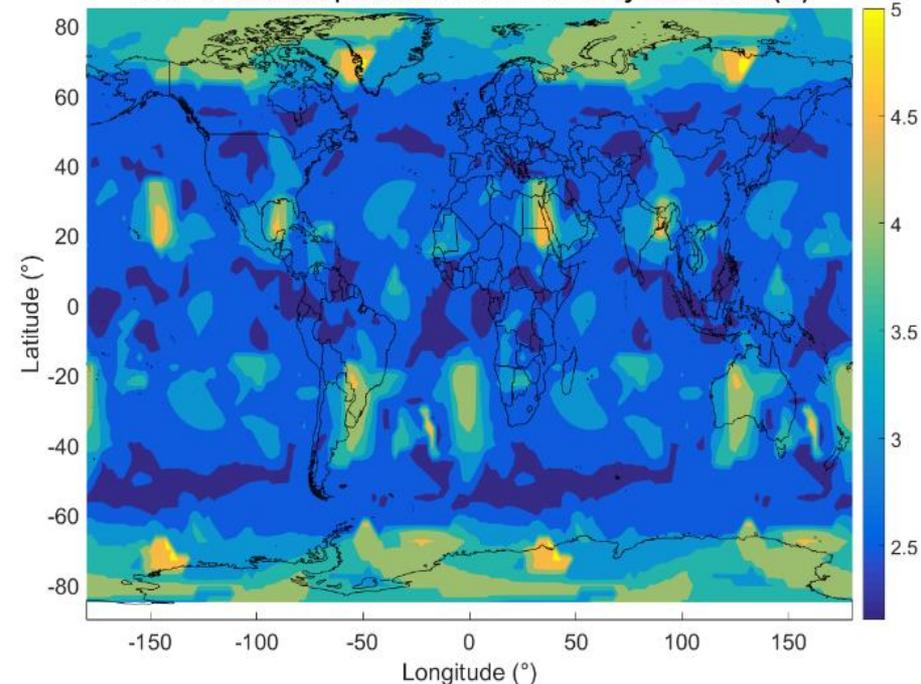
Same behaviors but values are higher than with biases induced by the satellite antenna.  
Logical looking at pseudorange bias models.

## 3) Highest position biases induced by the distortions

Maximum horizontal position biases induced by distortions (m)



Maximum vertical position biases induced by distortions (m)



	Max All lat
Horizontal position	2.4 m
Vertical position	5.1 m
3D position	5.5 m

Results obtained with  $|b_{max}| = 50 \text{ cm}$  on all pseudorange and the pseudorange bias sign is chosen to push the position bias value to his highest value as possible (they interfere in a constructive way).

➤ High position bias value (even in nominal conditions)

- I. Definitions related to pseudorange nominal bias
  1. Pseudorange nominal bias model
  2. Simulation concept
  3. Results: impact of pseudorange nominal biases on the position
  - 4. Conclusions and future works**

### 1) Conclusions (1/2)

- Proposition of a code pseudorange nominal bias definition in nominal conditions. 3 nominal bias components are introduced:
  - bias induced by the satellite antenna delay variation,
  - bias induced by the receiver antenna delay variation,
  - bias induced by distortions.
  
- Three models are proposed to characterize the three components constitutive of the total code pseudorange bias in nominal conditions considering a GPS L1 C/A civil aviation receiver.
  
- Impact on the position of pseudorange nominal biases characterized by the proposed models.

### 1) Conclusions (2/2)

Signal distortions have the highest impact on the 3D user position bias (up to 5.5 m) while the aggregate impact of the satellite and the user antenna delays does not exceed 2.6 m.

These nominal position bias values are relatively high and this because:

- Worst cases are looked at.
- Conservative models were retained to characterize biases induced by satellite antenna and receiver antenna delays.

### 2) Future works

- Test different bias models. In particular, the parameter  $b_{max}$  used to derive the impact of signal distortions on the position estimate could be defined more precisely instead of considering only a worst case (conspiring biases).
- Test more setup. For example a GPS (baseline) constellation with 24 satellites or/and with Galileo satellites.
- Instead of only looking at the worst case, study another observables. For example, world maps that give position biases that are obtained at 95%.
- Implement differential GNSS algorithms to assess the impact of pseudorange nominal biases on a differential user position.
- Look at other users than GPS L1 C/A signal only. For example Galileo signals users and/or dual frequency users and/or multiconstellation.

# Thank you for your attention

## Any questions?

[pagot@recherche.enac.fr](mailto:pagot@recherche.enac.fr)

[Haines et al., 2012] B. Haines, W. Bertiger, N. Desai, A. Sibois and J. Weiss, *IGS Workshop - Characterizing the GPS Satellite Antenna Phase- and Group-Delay Variations (poster)*, Jul. 2012.

[Harris, 2016] M. Harris, *Status for L1 GPS Group Delay Bounding Using Current DO-253 and DO-229 Airborne Multipath Error Model - presentation*, Oct. 2016.

[Macabiau et al., 2014] C. Macabiau, C. Milner, Q. Tessier, M. Mabileau, J. Vuillaume, N. Suard and C. Rodriguez, *Impact of Nominal Biases Bounding Techniques on Final ARAIM User Performance*, in Proceedings of ION ITM, San Diego, California, Jan. 2014.

[Raghuvanshi and Van Graas, 2015] A. Raghuvanshi and F. Van Graas, *Characterization of Airborne Antenna Group Delay Biases as a Function of Arrival Angle for Aircraft Precision Approach Operations*, in Proceedings of ION GNSS+, Tampa, Florida, Sep. 2015, pp. 3681–3686.

[RTCA, 2006] RTCA, *DO 301 - MOPS for GNSS Airborne Active Antenna Equipment for the L1 Frequency Band*, Dec. 2006.

## 2) How different biases can affect different pseudorange measurements?

### Error sources

#### Satellite

- Payload
- Antenna

#### ~~Propagation medium~~

Random residual errors

#### Receiver

- Antenna
- Radio-Frequency front-end.
- Tracking process

From the state-of-the-art (especially [Macabiau et al., 2014]), pseudorange nominal biases come from:

- The satellite and the receiver antennas and electronic devices that can generate signal **distortions**.
- The satellite antenna and electronic device and the receiver antenna that can generate signal **delays**.

Dependent upon the relative angles between the satellite antenna and the receiver antenna ( $N$  nadir,  $el$  elevation,  $az$  azimuth) in the receiver antenna coordinates system.

## 2) Simulations principles

Pseudorange biases depend on the relative positions between the satellites and the receiver antennas.



The impact of pseudorange biases depends on the relative position between the satellites and the receiver antennas.



For different relative position between satellites (used in the PVT estimation) and the receiver antennas, pseudorange biases and their impact on the position are estimated.

Nominal code pseudorange biases models described in the previous section are considered.

## 2) Delay induced by the satellite antenna

$$b^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + b_{dist}^{Xi}(N, el, az)$$

- Different consistent publications considered. Results from [Haines et al., 2012] were used. When results are not available for some PRNs, results from other PRNs are used (Block IIR-M for II-F).

## 2) Delay induced by the satellite antenna

$$b^{Xi}(N, el, az) = \mathbf{b}_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) + b_{dist}^{Xi}(N, el, az)$$

- Model initially given for L1/L2 iono-free measurements. It is assumed in this publication that it overbounds biases that would be obtained on L1 pseudorange measurements (conservative assumption). Additional studies are required to refine this model on L1.

$$b_{SV}^{L1/L2i} \approx 2.546 \times b_{SV}^{L1i} - 1.546 \times b_{SV}^{L2i}$$

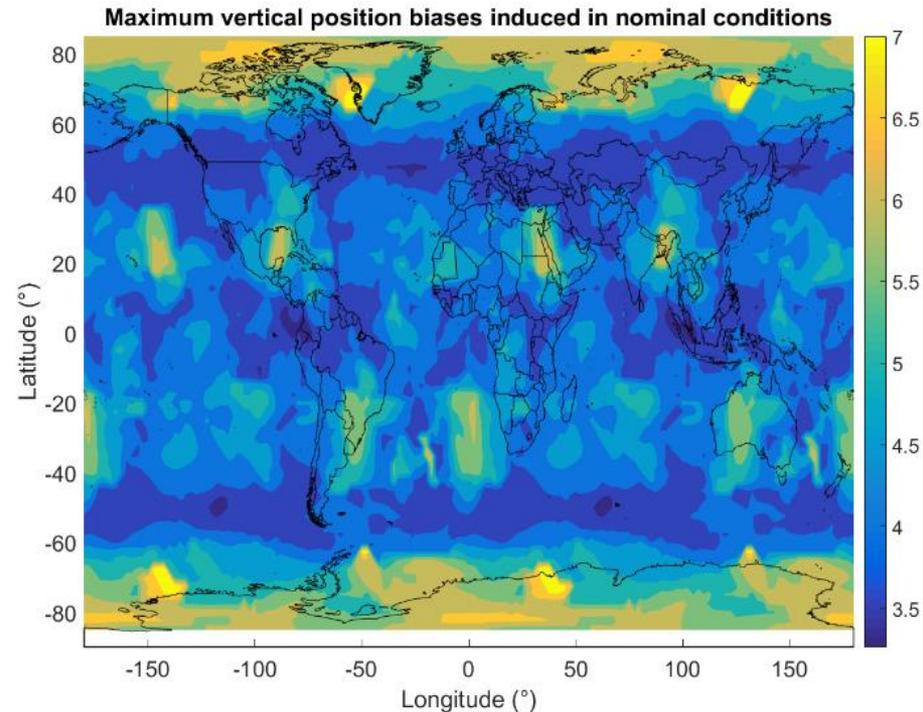
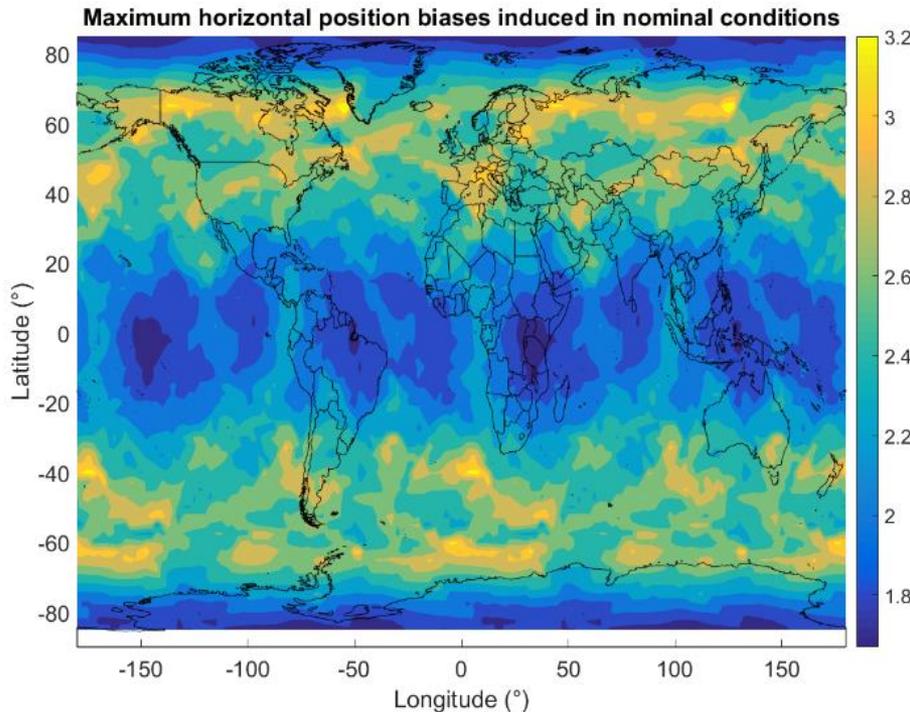
In the worst case:

$$\left| b_{SV}^{L1/L2i} \right| \approx 2.546 \times \left| b_{SV}^{L1i} \right| + 1.546 \times \left| b_{SV}^{L2i} \right|$$

Assuming that each  $b_{SV}^{L1i}$  and  $b_{SV}^{L2i}$  is random with a decreasing distribution around 0, we assume that there is a large probability that  $\left| b_{SV}^{L1i} \right| < \left| b_{SV}^{L1/L2i} \right|$ .

Same assumption used in [Macabiau et al., 2014] to justify that the model on  $b_{SV}^{L1/L2i}$  can be used conservatively to model  $b_{SV}^{L1/L5i}$  ( because  $2.546 + 1.546 = 4.092$  (L1/L2 combination) is higher than  $2.261 + 1.261 = 3.522$  )

## 4) Highest position biases induced by nominal deformations



	Maximum of maxima		
<b>Horizontal position</b>	1.5 m **	2.4 m **	3.4 m
<b>Vertical position</b>	2.6 m **	5.1 m **	7.5 m
<b>3D position</b>	2.6 m **	5.5 m **	8.1 m

Maximum impact on the position of nominal pseudorange biases induced by:

- delays caused by the satellite and the receiver antenna.
- distortions.
- **the summed effect of delays and distortions.**

## 3) Highest position biases induced by the distortions (1/2)

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = S(k) \times \begin{bmatrix} b^1 \\ \vdots \\ b^N \end{bmatrix} \quad S(k) = [H_{loc}^t H_{loc}]^{-1} H_{loc}^t(k) = \begin{bmatrix} S_{1,1}(k) & \dots & S_{1,N}(k) \\ S_{2,1}(k) & \dots & S_{2,N}(k) \\ S_{3,1}(k) & \dots & S_{3,N}(k) \\ S_{4,1}(k) & \dots & S_{4,N}(k) \end{bmatrix}$$

## 3) Highest position biases induced by the distortions (1/2)

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = S(k) \times \begin{bmatrix} b^1 \\ \vdots \\ b^N \end{bmatrix} \quad S(k) = [H_{loc}^t H_{loc}]^{-1} H_{loc}^t(k) = \begin{bmatrix} S_{1,1}(k) & \dots & S_{1,N}(k) \\ S_{2,1}(k) & \dots & S_{2,N}(k) \\ S_{3,1}(k) & \dots & S_{3,N}(k) \\ S_{4,1}(k) & \dots & S_{4,N}(k) \end{bmatrix}$$

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N S_{1,i}(k) b^i \\ \sum_{i=1}^N S_{2,i}(k) b^i \\ \sum_{i=1}^N S_{3,i}(k) b^i \\ \sum_{i=1}^N S_{4,i}(k) b^i \end{bmatrix}$$

Assuming that all measurements can be affected by a bias with an absolute amplitude  $b_{max}$ :

- the maximum position error along the horizontal plane is equal to:

$$b_{max} S_{tot\_H} = b_{max} \sqrt{\left( \sum_{i=1}^N |S_{1,i}| \right)^2 + \left( \sum_{i=1}^N |S_{2,i}| \right)^2}$$

- the maximum position error along the vertical is equal to:

$$b_{max} S_{tot\_V} = b_{max} \sum_{i=1}^N |S_{3,i}|$$