Determining GNSS Fault Probabilities

ITSNT: 16th November 2017 @ ENAC
Dr. Carl Milner
- **Foundations**
  
  

- **Bayes**

- **Estimated Rates**

- **Satellite Age**

- **Service Commitments**

- **Conclusions**
There are two types of fault with respect to the occurrence model [1][2]

Continuous time faults:
- clock run off / acceleration
- anomalous signal deformation
- code carrier divergence

Discrete time faults:
- ephemeris faults
- incorrect clock parameters
- incorrect health (during manoeuvre)

Models for both types of fault may be approximated by a Poisson process [3]
Continuous time faults occur at random times on the continuous time real domain.

The (abstract) instantaneous fault rate $\lambda(t)$ may vary with time.

A Non-Homogeneous Poisson Process (NHPP) \[4\]

\[
k = n(t_0 + d) - n(t_0) = \frac{(\int_{t_0}^{t_0+d} \lambda(t)dt)^k e^{-\int_{t_0}^{t_0+d} \lambda(t)dt}}{k!}
\]

Mean rate $r = \frac{1}{d} \int_{t_0}^{t_0+d} \lambda(t)\,dt$

Reduced to the Homogeneous Poisson Process (HPP) \[5\]:

\[
P(k|r, d) = \frac{(rd)^k e^{-rd}}{k!} \text{ conditioned on the unknown true rate } r
\]
Assertions used as the foundation for this modelling are [1]:

H1  Short term variations in λ are not observable

H2  \( \exists d^* > d \text{ s.t } r \geq \frac{1}{d^*} \int_{t_0}^{t_0 + d^*} \lambda(t) \, dt \)

H1 is reasonable since there are no measurements that can be taken that can be used to predict the likelihood of fault.

H2 is acceptable unless the true fault rate increases with age – see later slides.
Discrete time faults occur within the discrete navigation messages used for satellite orbit computation, clock correction etc

An Inhomogeneous Binomial Process is appropriate:

\[ \binom{n}{k} p_i^k (1 - p_i)^{n-k} \]

May approximate a Binomial Process by a Poisson Process (and IBP by NHPP) [6]

Requires \( n \) to be large and \( p \) to be small

\( \text{e.g } n = 100, p = 0.001 \quad \text{poisson} \quad 0.0905 \)
\( \quad \text{binomial} \quad 0.0906 \)
Combination of faults is therefore [3]:

\[ P(k|r, d) = \frac{(rd)^k e^{-rd}}{k!} \]

where:
- \( r \) is the mean rate of failure
- \( d \) duration of the applicable service history
- \( k \) number of observed faults of each type over \( d \)
- Foundations
- Bayes
- Estimated Rates
- Satellite Age
- Service Commitments
- Conclusions
Apply Bayes theorem [7]:

$$f(r|k) = \frac{P(k|r, d)f(r)}{P(k)}$$

Unknown prior $f(r)$ but can use a non-informative reference prior (Jeffrey’s prior [8]) which is invariant under the choice of parameterisation.

The $a$ posteriori conditional pdf is then [1][3]:

$$f(r|k) = \frac{d(rd)^{k-\frac{1}{2}}e^{-rd}}{\Gamma(k+\frac{1}{2})}$$

With expected value of $r|k$:

$$E(r|k) = \frac{k+\frac{1}{2}}{d}$$
In applications such as Advanced RAIM, we are concerned with the Probability of Hazardously Misleading Information under fault $F$

$$P(HMI, F|r) = P(HMI | F, r)P(F|r)$$

Can integrate over $r$, since for the Bayesian formulation $r$ has a distribution $f(r)$ [3]

$$P(HMI, F) = \int_{0}^{\infty} P(HMI|F, r)P(F|r)f(r)dr$$

Since the rate does not impact the left term of the integrand

$$P(HMI, F) = P(HMI|F) \int_{0}^{\infty} P(F|r)f(r)dr = P(HMI|F)E(P(F|r))$$

And so the probability of interest:

$$P(F|k) = MTTN.E(r|k) = MTTN.\frac{k + \frac{1}{2}}{d}$$
- Foundations
- Bayes
- Estimated Rates
- Satellite Age
- Service Commitments
- Conclusions
Wide satellite fault rate per hour

Estimated Rates

GLO 24SX 2011
6 years
1 wide fault [9]
\( P_{\text{const}} < 3.0 \times 10^{-5} \)
\( P_{\text{const}} < 5.0 \times 10^{-5} \)

GPS OCS+ 2007
10 years
0 wide faults
\( P_{\text{const}} < 6.0 \times 10^{-6} \)
\( P_{\text{const}} < 2.0 \times 10^{-5} \)

GPS FOC 1995
22 years
0 wide faults
\( P_{\text{const}} < 2.5 \times 10^{-6} \)
\( P_{\text{const}} < 8.0 \times 10^{-6} \)
If offline monitoring updated every say 6 or 12 months then can determine probability that the model holds by determining i.e. $P(k_{pred} \geq 2)$

$$P(k_{pred}) = \int_0^\infty P(k_{pred}|r, d_{pred})f(r|k)dr$$

Here $k$ is the observed i.e.
$k = 0$ for GPS
$k = 1$ for Glonass

Computes the risk that the ARAIM parameters do not remain valid
Narrow satellite fault rate:
- all constellation satellites treated identical
- block by block analysis

Note that the change in GPS service commitments in 2008 introduced a per satellite requirement [10] in place of an average satellite requirement [11]

We consider both 30 satellite constellation and a 24 satellite constellation and a block by block analysis

Note that for block based analysis, the duration is a cumulative sum of the individual satellites and not simply a product of the time duration
GLO 2009-2012
4 years
192 narrow faults
$P_{\text{const}} < 2.0 \times 10^{-4}$

GPS 2004-2012
8.5 years
28 narrow faults
$P_{\text{const}} < 2.0 \times 10^{-5}$

GAL 2018-2024
6 years
3 narrow faults
$P_{\text{const}} < 3.0 \times 10^{-6}$
GPS clock failures – average satellite age ≈11y

### Satellite Age

<table>
<thead>
<tr>
<th>Block</th>
<th>#Failures</th>
<th>Duration</th>
<th>MTBF</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIA</td>
<td>21</td>
<td>145y</td>
<td>7y</td>
<td>1.7e-5</td>
</tr>
<tr>
<td>IIR</td>
<td>6</td>
<td>105y</td>
<td>17.5y</td>
<td>7.0e-6</td>
</tr>
<tr>
<td>IIR-M</td>
<td>1</td>
<td>40y</td>
<td>40y</td>
<td>4.3e-6</td>
</tr>
<tr>
<td>IIF</td>
<td>0</td>
<td>3y</td>
<td>∞</td>
<td>1.9e-5</td>
</tr>
</tbody>
</table>

![Graph showing satellite age distribution](image)
Common objection with this approach is clock age – *bathtub curve effect* [15]

Note however, the service commitments are per satellite [10], not over the constellation [11]

Possible solution is:

1. assume initial operations cover early faults
2. cumulate the service history in the negative time direction
Accounting for age by cumulating age duration from infinity backwards and cumulating failures.

Only marginally more conservative.

**0-18 year old**
1.2×10⁻⁵ to 1.9×10⁻⁵

**19-22 year old**
2.1×10⁻⁵ to 1.7×10⁻⁴

<table>
<thead>
<tr>
<th>Age</th>
<th>Sats</th>
<th>NegCum Duration</th>
<th>NegCum Failures</th>
<th>$k + 1 + \frac{1}{2}/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>264</td>
<td>28</td>
<td>1.3×10⁻⁵</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>251</td>
<td>28</td>
<td>1.4×10⁻⁵</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>236</td>
<td>28</td>
<td>1.5×10⁻⁵</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>221</td>
<td>27</td>
<td>1.5×10⁻⁵</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>205</td>
<td>25</td>
<td>1.5×10⁻⁵</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>189</td>
<td>25</td>
<td>1.6×10⁻⁵</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>175</td>
<td>20</td>
<td>1.4×10⁻⁵</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>161</td>
<td>19</td>
<td>1.5×10⁻⁵</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>147</td>
<td>18</td>
<td>1.5×10⁻⁵</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>136</td>
<td>18</td>
<td>1.6×10⁻⁵</td>
</tr>
<tr>
<td>11</td>
<td>16</td>
<td>120</td>
<td>16</td>
<td>1.7×10⁻⁵</td>
</tr>
<tr>
<td>12</td>
<td>17</td>
<td>104</td>
<td>16</td>
<td>1.9×10⁻⁵</td>
</tr>
<tr>
<td>13</td>
<td>19</td>
<td>87</td>
<td>12</td>
<td>1.6×10⁻⁵</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>68</td>
<td>8</td>
<td>1.6×10⁻⁵</td>
</tr>
<tr>
<td>15</td>
<td>17</td>
<td>51</td>
<td>4</td>
<td>1.2×10⁻⁵</td>
</tr>
<tr>
<td>16</td>
<td>12</td>
<td>34</td>
<td>2</td>
<td>1.2×10⁻⁵</td>
</tr>
<tr>
<td>17</td>
<td>8</td>
<td>22</td>
<td>2</td>
<td>1.8×10⁻⁵</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>14</td>
<td>0</td>
<td>1.2×10⁻⁵</td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td>8</td>
<td>0</td>
<td>2.1×10⁻⁵</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>4.2×10⁻⁵</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>8.5×10⁻⁵</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1.7×10⁻⁴</td>
</tr>
</tbody>
</table>
- Foundations
- Bayes
- Estimated Rates
- Satellite Age
- Service Commitments
- Conclusions
Service Commitments

Service Commitments

Fault History

Bayesian Inference

Prior

Data

Trusted Parameters

Approach presented favours data over priors

ANSP could place more trust on the commitments
- Foundations
- Bayes
- Estimated Rates
- Satellite Age
- Service Commitments
- Conclusions
Address the question of how much trust to place in the service commitments or replace commitments with observed performance metrics

Use limited real data to make expanding window analysis using inference

Can be extended to outage probabilities (more data → better estimation)

Rely on as few assumptions as possible – although the system must be trusted not to degrade in the long term

Taking conservative assumptions results in practical fault rates that can meet the needs of ARAIM development (availability...?)

Authors view: should not carry forward assumptions of the past if no longer valid

i.e. \( P_{\text{const} \, GPS} = 0 \)

Is a long term solution to the requirements a preferred solution?
References

[7] Box and Tiao (1973) Bayesian Inference in Statistical Analysis
Questions...?